# Radioactive Decay

decay is the process in which an unstable nucleus spontaneously loses energy by emitting ionizing particles and radiation. This decay, or loss of energy, results in an atom of one type, called the **parent** nuclide, transforming to an atom of a different type, named the **daughter** nuclide.

The three principal modes of decay are called the alpha, beta and gamma decays.

These radioactive decays describe are fundamentally quantum processes, i.e. transitions among two quantum states.

Thus, the radioactive decay is statistical in nature, and we can only describe the evolution of the expectation values of quantities of interest, for example the number of atoms that decay per unit time.

If we observe a single unstable nucleus, we cannot know a priori when it will decay to its daughter nuclide. The time at which the decay happens is random, thus at each instant we can have the parent nuclide with some probability $p$ and the daughter with probability $1 - p$. This stochastic process can only be described in terms of the quantum mechanical evolution of the nucleus. However, if we look at an ensemble of nuclei, we can predict at each instant the average number of parent an daughter nuclides.

If we call the number of radioactive nuclei $N$, the number of decaying atoms per unit time is $dN/dt$. It is found that this rate is constant in time and it is proportional to the number of nuclei themselves:

$$\frac{dN}{dt} = -\lambda N(t)$$

The constant of proportionality $\lambda$ is called the **decay constant**.

where the RHS is the probability per unit time for one atom to decay. The fact that this probability is a constant is a characteristic of all radioactive decay. It also leads to the *exponential law of radioactive decay*:

$$N(t) = N(0)e^{-\lambda t}$$

We can also define the **mean lifetime**

$$\tau = 1/\lambda$$

and the **half-life**

$$t_{1/2} = \ln(2)/\lambda$$

which is the time it takes for half of the atoms to decay, and the **activity**

$$A(t) = \lambda N(t)$$

Since A can also be obtained as $|\frac{dN}{dt}|$ , the activity can be estimated from the number of decays $\Delta N$ during a small time $\delta t$ such that $\delta t \ll t_{1/2}$.



A common situation occurs when the daughter nuclide is also radioactive. Then we have a chain of radioactive decays, each governed by their decay laws. For example, in a chain $N_1 \rightarrow N_2 \rightarrow N_3$, the decay of $N_1$ and $N_2$ is given by:

$$dN_1 = -\lambda_1 N_1 dt, \qquad dN_2 = +\lambda_1 N_1 dt - \lambda_2 N_2 dt$$

Another common characteristic of radioactive decays is that they are a way for unstable nuclei to reach a more energetically favorable (hence stable) configuration. In $\alpha$ and $\beta$ decays, a nucleus emits a $\alpha$ or $\beta$ particle, trying to approach the most stable nuclide, while in the $\gamma$ decay an excited state decays toward the ground state without changing nuclear species.

# Alpha Decay

If we go back to the binding energy per mass number plot ($B/A$ vs. $A$) we see that there is a bump (a peak) for $A \sim 60 - 100$. This means that there is a corresponding minimum (or energy optimum) around these numbers. Then the heavier nuclei will want to decay toward this lighter nuclides, by shedding some protons and neutrons. More specifically, the decrease in binding energy at high $A$ is due to Coulomb repulsion. Coulomb repulsion grows in fact as $Z^2$, much faster than the nuclear force which is $\propto A$.

This could be thought as a similar process to what happens in the fission process: from a parent nuclide, two daughter nuclides are created. In the $\alpha$ decay we have specifically:

$${}^{A}_{Z}X_N \rightarrow {}^{A-}_{Z-}X'_{N-} \quad \alpha$$

where $\alpha$ is the nucleus of He-4: ${}_2^4He_2$.

The $\alpha$ decay should be competing with other processes, such as the fission into equal daughter nuclides, or into pairs including $^{12}C$ or $^{16}O$ that have larger $B/A$ then $\alpha$. However $\alpha$ decay is usually favored. In order to understand this, we start by looking at the energetic of the decay, but we will need to study the quantum origin of the decay to arrive at a full explanation.

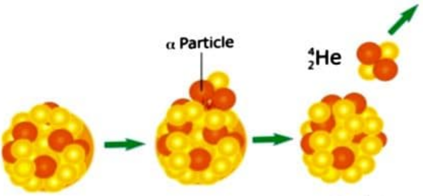


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Fig :Alpha decay schematics

A. Energetics

In analyzing a radioactive decay (or any nuclear reaction) an important quantity is $Q$, the net energy released in the decay: $Q = (m_X - m_{X'} - m_\alpha)c^2$. This is also equal to the total kinetic energy of the fragments, here $Q = T_{X'} + T_\alpha$ (here assuming that the parent nuclide is at rest).

When $Q > 0$ energy is released in the nuclear reaction, while for $Q < 0$ we need to provide energy to make the reaction happen. As in chemistry, we expect the first reaction to be a spontaneous reaction, while the second one does not

# Beta Decay

The beta decay is a radioactive decay in which a proton in a nucleus is converted into a neutron (or vice-versa). Thus $A$ is constant, but $Z$ and $N$ change by 1. In the process the nucleus emits a beta particle (either an electron or a positron) and quasi-massless particle, the **neutrino.**

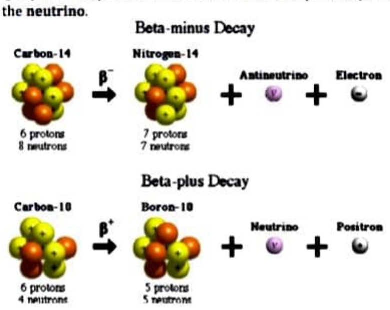


Fig:Beta decay schematics

There are 3 types of beta decay:

$$^{A}_{Z}X_{N} \rightarrow \, ^{A}_{Z+1}X'_{N-} \quad e^{-} \quad \bar{\nu}$$

This is the $\beta$- decay (or negative beta decay). The underlying reaction is:

$$n \rightarrow p + e^{-} + \nu^{-}$$

that corresponds to the conversion of a proton into a neutron with the emission of an electron and an anti-neutrino. There are two other types of reactions, the $\beta^{+}$ reaction,

$$^{A}_{Z}X_{N} \rightarrow \, ^{A}_{Z-1}X'_{N} \quad e^{+} \quad \nu \quad \Longleftrightarrow \quad p \rightarrow n \quad e^{+} \quad \nu$$

which sees the emission of a positron (the electron anti-particle) and a neutrino;



And the electron capture:

$$^{A}_{Z}X_{N} + e^{-} \rightarrow \, ^{A}_{Z-1}X'_{N} \quad \nu \quad \Longleftrightarrow \quad p + e^{-} \rightarrow n \quad \nu$$

The beta decay is the radioactive decay process that can convert protons into neutrons (and vice-versa). We will study more in depth this mechanism, but here we want simply to point out how this process can be energetically favorable, and thus we can predict which transitions are likely to occur, based only on the SEMF.
For example, for $A = 125$ if $Z < 52$ we have a favorable $n \rightarrow p$ conversion (beta decay) while for $Z > 52$ we have $p \rightarrow n$ (or positron beta decay), so that the stable nuclide is $Z = 52$ (tellurium).

A. Conservation laws

As the neutrino is hard to detect, initially the beta decay seemed to violate energy conservation. Introducing an extra particle in the process allows one to respect conservation of energy. The Q value of a beta decay is given by the usual formula:

$$Q_{\beta^-} = [m_N(^{A}X) - m_N(^{A}_{Z+1}X') - m_e]c^2.$$

Using the atomic masses and neglecting the electron's binding energy.

The kinetic energy (equal to the Q) is shared by the neutrino
and the electron (we neglect any recoil of the nucleus ).

Notice that the neutrinos also carry away angular momentum. They are spin-1/2 particles, with no charge (hence the name) and very small mass. For many years it was actually believed to have zero mass. However it has been confirmed that it does have a mass in 1998.
Other conserved quantities are:

- **Momentum:** The momentum is also shared between the electron and the neutrino. Thus the observed electron momentum ranges from zero to a maximum possible momentum transfer. - **Angular momentum** (both the electron and the neutrino have spin 1/2)
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- Parity? It turns out that parity is not conserved in this decay. This hints to the fact that the interaction responsible violates parity conservation (so it cannot be the same interactions we already studies, e.m. and strong interactions)
- - **Charge** (thus the creation of a proton is for example always accompanied by the creation of an electron)
- **Lepton number** is conserved.
- **Baryon number** is conserved.

# Gamma Decay

In the gamma decay the nuclide is unchanged, but it goes from an excited to a lower energy state. These states are called isomeric states. Usually the reaction is written as:

$$^{A}_{Z}X^{*}_{N} \longrightarrow \, ^{A}_{Z}X_{N} \quad \gamma+$$

where the star indicate an excited state. We will study that the gamma energy depends on the energy difference between these two states, but which decays can happen depend, once again, on the details of the nuclear structure and on quantum-mechanical selection rules associated with the nuclear angular momentum.