Addition Theorem of Probability

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08/07/2020

Economics Online Class
Subject: Statistics
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08.07.2020

Addition theorem of Probability

If A and B are any two events in a sample space, S, then the probability of occurance of at least one of the events A and B is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

where,
$P(A \cup B)$ = Probability of occurance of A or B
$P(A)$ = Probability of occurance of event A
$P(B)$ = Probability of occurance of event B
and $P(A \cap B)$ = Probability of occurance of both A and B

If A and B are mutually exclusive events, then,

$$P(A \cup B) = P(A) + P(B)$$

[∵ for mutually exclusive events, $A \cap B = \phi$ (empty set)
hence $P(A \cap B) = 0$]

If A, B and C are three events in a sample space S, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

Example. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of its being a spade or a king.

Let $P(A)$ = Probability of the card being a spade
and $P(B)$ = Probability of the card being a king.
and $P(A \cap B)$ = Probability of the card being king of spade.

$\therefore P(A) = \frac{13}{52} = \frac{1}{4}$ (∵ there are 13 spade in a pack)

$P(B) = \frac{4}{52} = \frac{1}{13}$ (∵ there are 4 kings in a pack)

and $P(A \cap B) = \frac{1}{52}$ (∵ there is only 1 king of spade)

∴ The required probability (i.e. the probability of the card being king or spade) is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or, $P(A \cup B) = \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13}$.

Some numericals on addition theorem are given below:

Numericals on Addition theorem of probability

Question ① The odds in favour of winning the race for three horses are 1:2, 2:5 and 1:7 respectively. What is the probability that either of them will win the race?

Answer Let A, B and C be the events of three different horses winning the race.

Given, $P(A) = \frac{1}{1+2} = \frac{1}{3}$; $P(B) = \frac{2}{2+5} = \frac{2}{7}$ and $P(C) = \frac{1}{1+7} = \frac{1}{8}$

Since A, B and C are mutually exclusive events;

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$= \frac{1}{3} + \frac{2}{7} + \frac{1}{8} = \frac{56+48+21}{168} = \frac{125}{168}$$

Question ② What is the probability of getting 4 or 7 or 12 in the throw of two dice?

Here, the random experiment is tossing of two dice.

Sample space; $n(S) = 36$.

Let A = event of getting a total of 4.

B = event of getting a total of 7.

C = event of getting a total of 12.

Now,

$A = \{(1,3)(3,1),(2,2)\}$

$B = \{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$

$C = \{(6,6)\}$

$\therefore n(A) = 3$, $n(B) = 6$ and $n(C) = 1$

Hence, $P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

and $P(C) = \frac{n(C)}{n(S)} = \frac{1}{36}$

Since, the events are mutually exclusive, the probability of getting 4 or 7 or 12 in a throw of two dice is

$P(A \cup B) = P(A) + P(B) + P(C) = \frac{1}{12} + \frac{1}{6} + \frac{1}{36} = \frac{3+6+1}{36} = \frac{10}{36} = \frac{5}{18}$.