

Numericals on Probability

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A question of mathematics is given to four students A, B, C and D whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$ and $\frac{1}{3}$ respectively. Find the probability of the question being solved.

Given

$P(A)$ = probability of A solving the question = $\frac{1}{2}$

$P(B)$ = probability of B solving the question = $\frac{3}{4}$

$P(C)$ = probability of C solving the question = $\frac{1}{4}$

$P(D)$ = probability of D solving the question = $\frac{1}{3}$.

Let A' , B' , C' and D' be the probabilities of A, B, C and D not solving the question.

$$\therefore A' = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2} ; B' = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}.$$

$$C' = 1 - P(C) = 1 - \frac{1}{4} = \frac{3}{4} \text{ and } D' = 1 - P(D) = 1 - \frac{1}{3} = \frac{2}{3}.$$

Let the probability of the question being solved be E.

$$\therefore P(E) = 1 - P(\text{none being able to solve the problem})$$

$$\text{or } P(E) = 1 - [P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})]$$

$$= 1 - [P(A) \cdot P(B) \cdot P(C) \cdot P(D)]$$

$$= 1 - \left[\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{2}{3} \right]$$

$$= 1 - \left[\frac{1}{16} \right] = \frac{15}{16}.$$

Therefore, the probability of at least one person (at least) solving the problem is $\frac{15}{16}$.

A speaks truth in 60% cases and B in 70% cases. What is the probability of their contradicting each other?

Let

$P(A)$ = Probability of A speaking the truth = $\frac{60}{100} = 0.6$ (Given)
and $P(B)$ = Probability of B speaking the truth = $\frac{70}{100} = 0.7$ (Given)

$\therefore P(\bar{A})$ = Probability of A not speaking the truth
= $1 - 0.6 = 0.4$

and $P(\bar{B})$ = Probability of B not speaking the truth
= $1 - 0.7 = 0.3$

\therefore The probability that A and B would contradict each other is $P(E)$:

$$\begin{aligned} \text{or, } P(E) &= P(A \cap \bar{B}) \cup (B \cap \bar{A}) \\ &= \{P(A) \cdot P(\bar{B})\} \cup \{P(B) \cdot P(\bar{A})\} \\ &= \{(0.6)(0.3)\} + \{(0.7)(0.4)\} \\ &= 0.18 + 0.28 \\ &= 0.46 \quad \text{or } 46\% \end{aligned}$$

Therefore, the probability that A and B would contradict each other is 0.46. In other words A and B are likely to contradict each other in 46% cases.

A bag contains 3 white and 2 black balls.
Another bag contains 5 white and 3 black balls.
If a bag is chosen at random, what a ball is taken out, what is the probability of the ball being white?

Let

A_1 = probability of choosing the first bag.

A_2 = probability of choosing the second bag.

B = probability of drawing a white ball.

∴ Probability of choosing a white ball from the first bag =

$$P(B/A_1) = \frac{3}{3+2} = \frac{3}{5}$$

Probability of drawing a white ball from the second bag =

$$P(B/A_2) = \frac{5}{5+3} = \frac{5}{8}$$

$$P(A_1) = \frac{1}{2} = P(A_2)$$

∴ The desired probability is

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B)$$

$$= P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)$$

$$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{5}{8}$$

$$= \frac{3}{10} + \frac{5}{16} = \frac{24+25}{80} = \frac{49}{80}$$