Fourier transform (contd.)

- Convolution product of $f(x)$ & $g(x)$

$$f*g = \int_{-\infty}^{\infty} f(u)\cdot g(x-u)\,du.$$

- <u>Convolution Theorem</u> Given $f(x)$ and $g(x)$. Then

$$\mathcal{F}\{f(x)*g(x)\} = \mathcal{F}\{f(x)\}\cdot\mathcal{F}\{g(x)\}.$$

i.e. Fourier transform of convolution product of $f(x)$ and $g(x)$ is the product of their Fourier transforms.

<u>Proof.</u> $$\mathcal{F}\{f(x)*g(x)\} = \int_{-\infty}^{\infty}\{f(x)*g(x)\}\,e^{-isx}\,dx$$

$$= \int_{x=-\infty}^{\infty}\left[\int_{u=-\infty}^{\infty} f(u)\cdot g(x-u)\,du\right]e^{-isx}\,dx$$

Since the limits of integrals all constants, on changing the order of integrals, limits won't change.

Therefore,

$$\mathcal{F}\{f(x)*g(x)\} = \int_{u=-\infty}^{\infty} f(u)\left[\int_{x=-\infty}^{\infty} g(x-u)\,e^{-isx}\,dx\right]du$$

$$= \int_{u=-\infty}^{\infty} e^{-isu}\,f(u)\left[\int_{x=-\infty}^{\infty} e^{-is(x-u)}\,g(x-u)\,d(x-u)\right]du$$

$$= \mathcal{F}\{f(x)\}\cdot\mathcal{F}\{g(x)\}.$$