

# Binomial Distribution

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Binomial distribution is probability distribution expressing the probability of one set if dichotomous alternatives, i.e. success or failure. This type of process, also known as the Bernoulli process, has wide application in Economics.

A Bernoulli process has the following features:

- 1 An experiment is performed under the same condition for a fixed number of trials  $n$ .
- 2 In each trial, there are only two possible outcomes, success or failure.
- 3 The trials are statistically independent. Examples of Bernoulli trials:
  - 1 tossing a coin having two outcomes head or tail
  - 2 rolling a dice when only one number is success and others are failures.
  - 3 an opinion poll with two outcomes yes or no.

Here are some numericals based on binomial probability.

Q. A die is thrown 5 times. What is the chance that an even number will come up exactly three times?  
Let

E be the event of occurrence of an even number  
and S be the sample space

$$\therefore S = [1, 2, 3, 4, 5, 6] \therefore n(S) = 6$$

$$\text{and } n(E) = [2, 4, 6] \therefore n(E) = 3$$

$\therefore$  Probability  $P$  of occurrence of an even number (or event E) in one trial

$$P = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$\therefore$  Probability of non-occurrence of the event  $q = 1 - \frac{1}{2} = \frac{1}{2}$

Number of trials = 5 (given)

$\therefore$  The probability of occurrence of an even number exactly 3 times in 5 trials is

$$P(X=3) \text{ or } P(3) = {}^5C_3 P^3 q^2$$

$$\left[ \because P(r) = {}^nC_r P^r q^{n-r} \right]$$

$$\text{or } P(3) = \frac{5!}{3!(5-3)!} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$\text{or } P(3) = \frac{5 \times 4 \times 3!}{3! \cdot 2!} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$\text{or } P(3) = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = \frac{5}{16}$$

A bag contains 4 white and 6 black balls. A ball is picked at random and replaced. If this is repeated 4 times, what is the probability that 3 white and one white ball will be drawn in these 4 trials?

Given

Number of black balls = 6 (Six)

Number of white balls = 4 (Four)

Number of trials = 4

Probability that a black ball will be drawn in one trial  $p = \frac{n(E)}{n(S)} = \frac{6}{10} = \frac{3}{5}$

$\therefore$  Probability that a white ball will be drawn in one trial  $q = 1 - \frac{3}{5} = \frac{2}{5}$

$\therefore$  Required probability

$$P(X=3) \text{ or } P(3) = {}^4C_3 (p^3 \cdot q^1)$$

$$[P(r) = {}^nC_r p^r q^{n-r}]$$

$$= 4 \left( \frac{3}{5} \right)^3 \left( \frac{2}{5} \right)^1$$

$$= 4 \cdot \frac{27}{125} \cdot \frac{4}{5} = \frac{216}{625}$$